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Analysis of Earth-Uranus Direct-Transfer Trajectory for Optimal Delta-V Using Lambert's Problem

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Abstract: The Ice Giants may become a sought-after destination in the coming decades as researchers aim to have a better awareness of our Solar system- its origins and growth. The interplanetary trajectory optimization is an important aspect of the analysis of a mission to Uranus. This study investigates possible interplanetary paths to Uranus in the 2022-2030 timeframe. It provides a preliminary estimate of fuel consumption in units of ΔV for various mission durations. A variety of approaches can be used to travel from Earth to another planet. It is conceivable to use a direct transfer route with two engine burns, one at a parking orbit around the Earth and the other to capture around the target planet. This article emphasizes a direct transfer trajectory analysis towards Uranus using Lambert's problem. Different lambert arcs were considered for the direct transfer. Variations of excess velocities at arrival and departure for various time-of-flight were obtained. The ceiling of the time-of-flight was fixed as 16.5 years by performing a Hohmann transfer. The minimum ΔV was obtained for various time-of-flight ranging from 8.5 years to 16.5 years. The ideal ΔV obtained during the fixed timeframe lies between 6.87 km/s and 7.98 km/s. The minimum value of ΔV was observed for the time-of-flight of 13.5 years.

Keywords: Direct Transfer Trajectory, Lambert's Problem, Patched-Conic Method, Earth-Uranus Mission, Optimal Delta-V, Interplanetary Mission

1. Introduction

Since the dawn of the space age, several missions have been flown to explore the outer planets. Early flyby missions of Jupiter and Saturn created new questions about the origin of our solar system and the possibility of life outside of Earth. Multiple flyby missions, as well as dedicated orbital missions, have since been flown to explore these planets in greater detail. Meanwhile, the only data points we have for Uranus and Neptune are a single flyby of each provided by the Voyager 2 spacecraft. 30 years ago, a fleeting sight of Uranus and Neptune revealed inexplicable worlds that demand dedicated missions to understand their mysteries. A comprehensive examination of Uranus's rings, satellites, and other features will teach us a lot about our solar system, as well as ice giant exoplanets.

To investigate Uranus' atmosphere, the Oceanus mission concept exploited the Saturn-Uranus trajectory that would launch in 2028 by aligning the two planets and arrive at Uranus in 2040 [1]. Sayanagi et al. explain the feasibility of employing a multi-probe mission to giant planets [2]. Even though high-heritage mission architectures like QUEST [3] have been designed to gain a deeper knowledge of the solar system's magnetospheres and beyond. It is not difficult to send a spacecraft to Uranus, but flight time is one of the most difficult challenges for an ice giant mission. The key is to reduce this time and to send and operate spacecraft in the darkness of the outer solar system.

Many interplanetary trajectory analyses toward Uranus using gravity assist, and Deep Space Maneuvers (DSM) [4-8], have been studied for the optimization of ΔV. To optimize ΔV for a mission, several genetic algorithms are in existence. One is a case learning-based differential evolution algorithm that performed well on the GTOP benchmark in a reasonable amount of time [9]. Second is the algorithms proficient in reaching objectives located in the inner and outer solar system and the orbits of the targets that are extremely inclined to the ecliptic plane [10]. Automated design algorithms can be applied to several contemporary issues for the upcoming 60 years, like a large set of latent paths toward the exterior solar system, and a lot more. Likewise, trajectory optimization is done using various software platforms such as GMAT [11], GALOMUSIT [12], ESA's PYGMO and PYKEP [13], MALTO [14], and JPL's MONTE [15]. Iorfida discussed the optimization of impulsive transfer trajectories using primer vector theory to develop a novel approach in the field of mid-course corrections with DSM [16]. Woo & Cupples employed a combination of genetic algorithm and calculus of-variations optimization program to engender a commission to Neptune with directed attributes [17].

Torla & Peet described a direct transfer from an Apex anchor to Mars [18]. Here the optimum ΔV was attained for an array of preliminary requirements and TOF restraints, using iterative approaches based on a distinction of Lambert's problem. To provide ΔV related to different mission durations, researchers considered two different multiple gravity-assist interplanetary trajectory schemes towards Uranus and Neptune in the 2025-2100 timeframe [13]. Tang & Conway demonstrated the approach of direct collocation with nonlinear programming for the optimization of interplanetary transfers, including the departure and arrival phases of flight [19].

Research shows that trajectory Optimization of Earth–Saturn and Earth–Jupiter Missions are solved in two phases [20]: i) with a zero-DSM stage, and a suboptimal flyby structure and ii) with a multi gravity-assist using DSMs and a fixed flyby sequence. A method based on the pseudo-state technique was proposed, giving four different transfer trajectory proposal alternatives possible for an interplanetary satellite mission [21]. This was with fixed inclinations of parking orbits. Hargraves & Paris combined mathematical programming with an embedded collocation method for trajectory optimization [22]. An Evolutionary Neuro Control solution as suggested by Dachwald can be employed as a preliminary assumption for customary trajectory optimization methods [23]. They were intended for a broad range of minimum ΔV questions. Likewise, low-thrust trajectory optimization was carried out by merging artificial neural networks and evolutionary algorithms based on machine learning perspectives, which reduced the transfer time by 74% [24].

Most current low thrust path optimizers [25, 26] remain sophisticated and challenging to integrate into the smaller spacecraft system prototypes utilized for synchronized work. Furthermore, many of them are unable to accommodate mission planning or discrete operations. Yam et al. used the LTGA trajectory design methodology to learn rendezvous missions with and without gravity assist in the launch years from 2014 to 2025 [6]. This approach was used to carry spacecraft to the outside planets in a sensible time of flight. From this, it is concluded that missions to Neptune or Pluto without gravity assist are impracticable, but it is feasible to have a mission towards Uranus devoid of gravity assist. Because they often rely on gradient-based methodologies, they should include optimization as part of the solution. ΔV trajectory optimization methods that can search through multi-objective data are scarce.

Taking into consideration all the above limitations and the possibility of an Earth-Uranus mission without gravity assist, the trajectory optimization is decided to be carried out using the direct transfer method. In this work, the authors investigate the possibility of direct interplanetary transfer from Earth to Uranus for a specific set of launch windows. The objective is to identify the optimum ΔV for various time-of-flight (TOF) in the departure timeframe 2022–2030.

2. Mathematical Method

At the beginning of its journey towards Uranus, the spacecraft is placed in a departure geostationary transfer parking orbit with a perigee altitude of 185 km. To begin its journey towards Uranus, the spacecraft is placed in a geostationary parking orbit with a perigee altitude of 185 km. At arrival, the spacecraft must be positioned in a highly elliptical polar trajectory parking orbit with an eccentricity of 0.90 and a perigee altitude of 2500 km. The real-time position and velocity vectors of the planets are considered. It is assumed that the mass distribution of the planets is radially symmetric and that disturbing forces such as solar radiation pressure, electromagnetic forces, aerodynamic forces, etc. acting on the spacecraft are negligible.

To perform the trajectory optimization analysis, necessary data must be acquired that follows a sequence of strides. The study is conducted to calculate the position vector of the planets and TOF using the patched conic method. The essential direct transfer trajectory has been established for the given TOF using Lambert's problem.

The following are the steps to get the required data for the trajectory analysis:

Step 1: Find the approximate position of Earth and Uranus from their Keplerian orbital elements using Eqs. (1)-(7). Classical and alternate orbital elements of the planets at epoch J2000 and their rates are given in table 1.

For a particular Julian Ephemeris Date, T_{eph}, the coordinates of Uranus can be found by the following steps:

For each of the 6 orbital elements, find the $a = a_0 + \dot{a}T$, where T is the number of centuries past J2000, that is,

$$T=(T_{eph}-2451545.0)/36525 \tag{1}$$

a) Find argument of perigee ω and the mean anomaly M,

$$\omega = \overline{\omega} - \Omega; M = L - \overline{\omega} + bT^2 + c\cos(fT) + s\sin(fT) \tag{2}$$

b) From the mean anomaly find the eccentric anomaly,

$$M = E - e^{*}\sin E, \text{ where } e^{*} = 57{:}29578\, e \quad (3)$$

c) Find the heliocentric coordinates of Uranus in its orbital plane, r', x', from focus to perihelion.

$$x' = a\cos E - e;\ y' = a\sqrt{1-e^2}\,\sin E;\ z' = 0 \quad (4)$$

Step 2: Obtain the solution of Kepler's equation by the Newton-Raphson method [28, 29].

$$M = E - e^{*}\sin E, \quad (5)$$

given the mean anomaly and e^{*}

$$E_0 = M + e^{*}\sin M, \quad (6)$$

and the three following equations are iterated:

$$\Delta M = M - (E_n - e^{*}\sin E_n);$$

$$\Delta E = \frac{\Delta M}{(1 - e\cos E_n)};\ E_{n+1} = E_n + \Delta E \quad (7)$$

Step 3: Calculation of the TOF and the heliocentric velocity vector of a planet at a given time.

To study the Earth-Uranus mission the patched conic interplanetary trajectory is used. It gives a trajectory with the sun at the center and intersecting the two planets at positions P1 and P2 as shown in Figure 1. Eventually, the spacecraft is taken from the SOI of the earth to the SOI of Uranus. The transfer time of this mission is obtained from Eq. (8) and is considered the ceiling for the TOF of Lambert's trajectory. The heliocentric velocities of the transfer orbit are calculated (Eqs. (9) & (10)) at the SOI. Thus, the velocities at infinitude are identified and utilized to establish planetocentric departure and arrival trajectories at Earth as well as Uranus. In this approach, the three conic sections, with the sun at the center and the further two concentrated on the planets in question (here Earth and Uranus), are 'patched' together [27, 29, 30].

$$\text{TOF} = \frac{\pi}{\sqrt{\mu_{sun}}}\left(\frac{r_1+r_2}{2}\right)^{3/2} \quad (8)$$

$$V_E = \left(\sqrt{\frac{2\mu_{sun}}{r_1} - \frac{\mu_{sun}}{a_e}}\right) \quad (9)$$

$$V_U = \left(\sqrt{\frac{2\mu_{sun}}{r_2} - \frac{\mu_{sun}}{a_u}}\right) \quad (10)$$

Step 4: Solution of Lambert's problem from position vector of planets and TOF [20].

Following a series of procedures yields the simplest option for developing a direct transfer trajectory to other planets. Find the departing planet's location r_1 and r_2 at times t_1 and t_2 respectively. Initially, at time t_1 the spacecraft is at P_1 and finally, at time t_2 it reaches position P_2. To put it another way, the journey time is $\Delta t = t_2 - t_1$. The first two steps can be determined using planetary ephemeris. The solution to Lambert's problem yields the third step. In the Lambert solver, a plane with three points, r_1, r_2, and M, is characterized where M is the Sun, and the connecting trajectory lies in it. It assumes a semi-major axis of this trajectory to find an elliptic path that touches both r_1 and r_2. An iteration on the semi-major axis is performed till the transfer time is precisely Δt because this apparent semi-major axis does not give an accurate Δt from r_1 to r_2. Since the Hohmann transfer time is found to be 16.5 years, the range of TOF taken for Lambert's trajectory is from 15.5 to 8.5 years. The algorithm of Lambert's problem is shown in figure 1.

Table 1. *Orbital elements and their time derivatives, for the mean ecliptic and equinox of J2000 [29].*

	a (AU) **$\dot{a}$ (AU/Century)**	**e** **$\dot{e}$ (1/Century)**	**i (degree)** **$\dot{i}$ (°/Century)**	**Ω (degree)** **$\dot{\Omega}$(°/Century)**	**$\bar{\omega}$ (degree)** **$\dot{\bar{\omega}}$(°/Century)**	**L (degree)** **$\dot{L}$(°/Century)**
Earth	1.00000261 0.00000562	0.01671123 -0.00004392	-0.00001531 -0.01294668	0.0 0.0	102.93768193 0.32327364	100.46457166 35,999.37244981
Uranus	19.18916464 -0.00196176	0.04725744 -0.00004397	0.77263783 -0.00242939	74.01692503 0.04240589	170.95427630 0.40805281	313.23810451 428.48202785

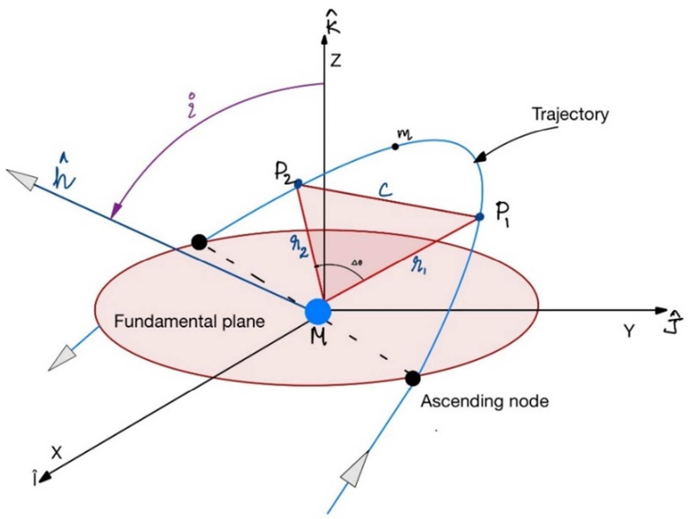

Figure 1. *Lambert's Problem.*

It's worth stating that the ellipse's period and the eccentricity do not affect the specific mechanical energy. Therefore, for any date of departure on one planet and arrival period on the other planet, a trajectory can be found for this mission. However, there are 2 outputs of the Lambert solver that is departure and arrival hyperbolic excess velocity. Finally, the total ΔV for various TOF from 15.5 to 8.5 years for the departure year 2022 – 2030 is obtained to examine the influence on ΔV.

Determining spacecraft orbit based on planetary positions and flight time is presented. The trajectory optimization towards Uranus to study the effect of ΔV for TOF varying from 15.5 to 8.5 years for the departure year 2022 – 2030 is performed using the direct transfer method. The objective is to maintain the ΔV as much low as possible (in this work less than 8 km/s) for all the TOF to reduce the fuel consumption.

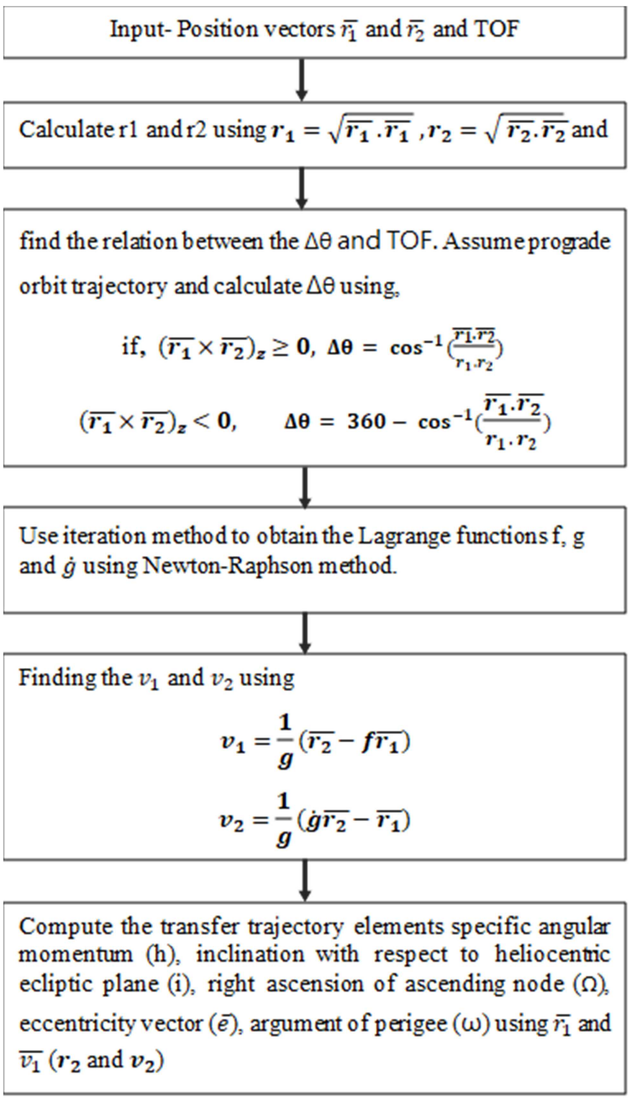

Figure 2. *Algorithm of Lambert's problem.*

It is assumed that the engine burn is performed at the pericentre of the parking orbit. This is done to take advantage of the fact that the velocity in periapsis is at its highest value within the orbit. To calculate the ΔV needed at departure or arrival, the characteristics of the parking orbits need to be known.

Every so often, the hyperbolic excess velocities and the total ΔV should be minimized for various TOFs. To get the trajectory with an optimum total ΔV, repeat the algorithm by changing the inputs of Lambert's problem and find the ideal departure and arrival times t1 and t2 for various TOFs. For each TOF, different departure dates between 2022 and 2030 are considered for which the total ΔV has been obtained. Interestingly the ΔV attained was the least for July or August for the entire departure year time frame.

3. Results and Discussion

3.1. Earth-Uranus Direct Transfer Trajectory Analysis - V_∞ Variation

Figures 3-10 show the effect of hyperbolic excess velocity at departure and arrival in the departure year 2022 to 2030 for the TOF ranging from 15.5 to 8.5 years.

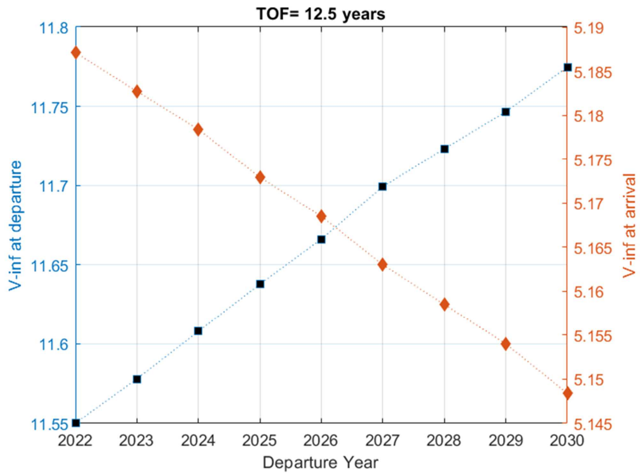

Figure 3. *V_∞variation vs departure year for TOF= 15.5 years.*

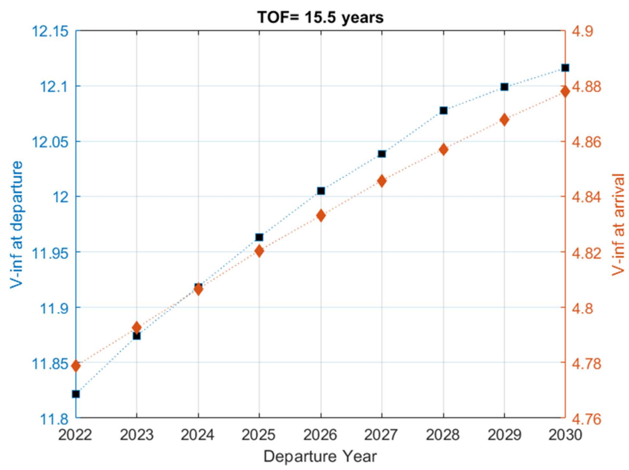

Figure 4. *V_∞variation vs departure year for TOF= 14.5 years.*

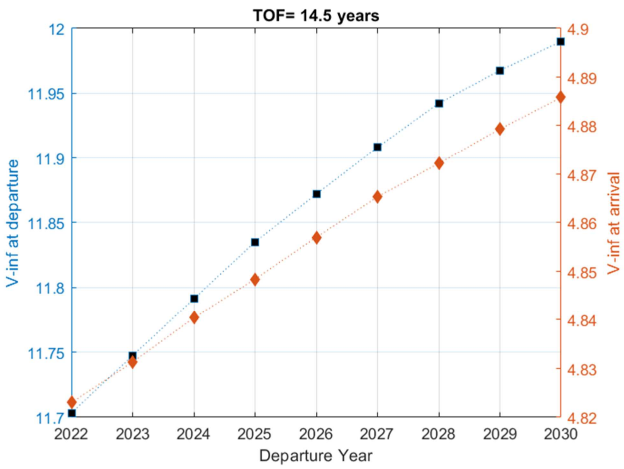

Figure 5. *V_∞ variation vs departure year for TOF= 13.5 years.*

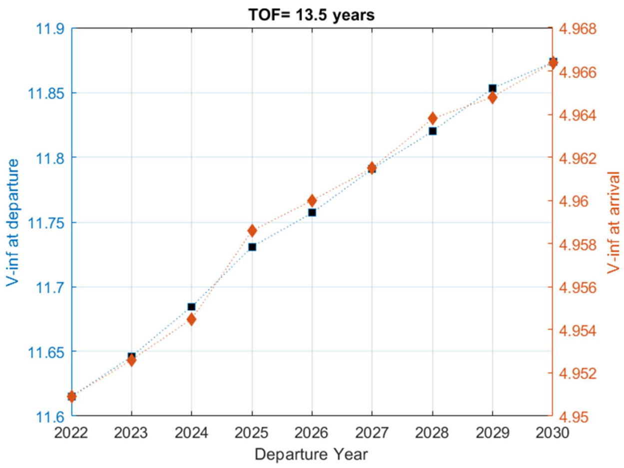

Figure 6. *V_∞ variation vs departure year for TOF= 12.5 years.*

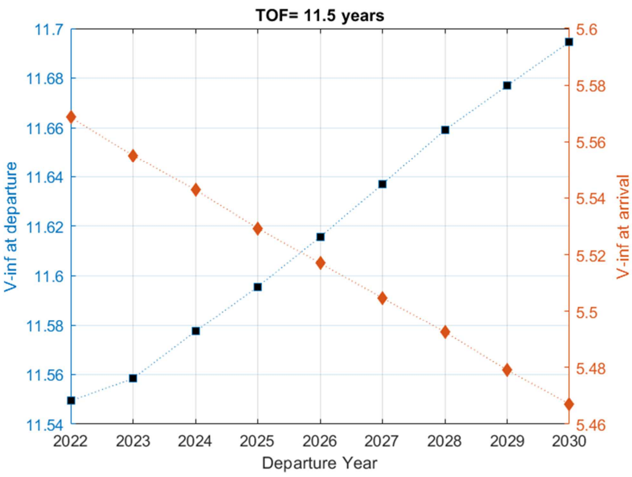

Figure 7. *V_∞ variation vs departure year for TOF= 11.5 years.*

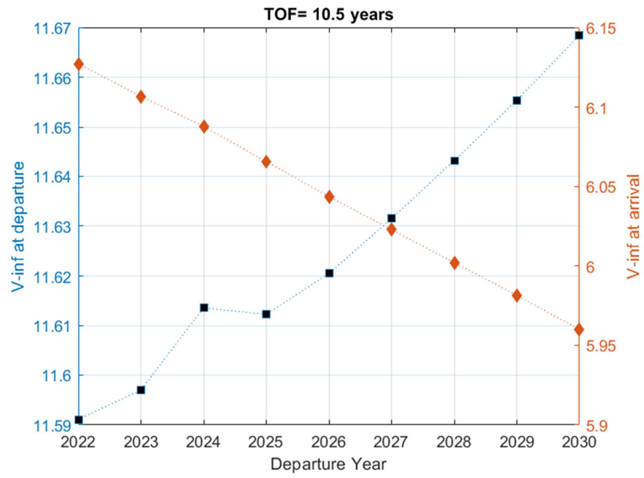

Figure 8. *V_∞ variation vs departure year for TOF= 10.5 years.*

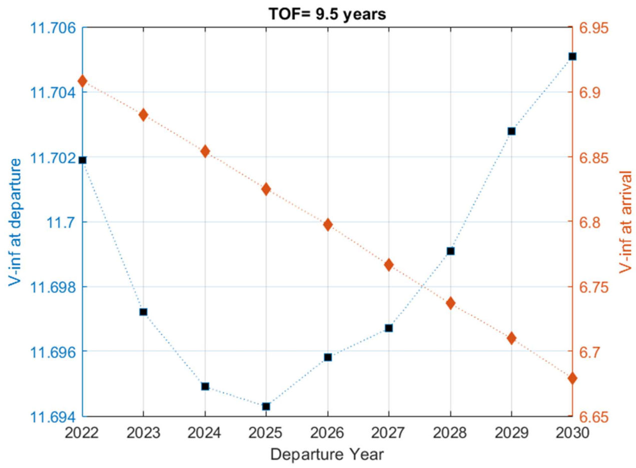

Figure 9. *V_∞ variation vs departure year for TOF= 9.5 years.*

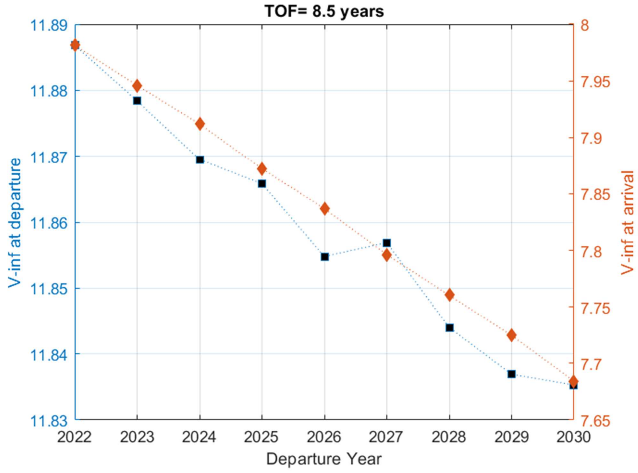

Figure 10. *V_∞ variation vs departure year for TOF= 8.5 years.*

3.2. Earth-Uranus Direct Transfer Trajectory Analysis - ΔV Variation

Figures 11-19 show the effect of total ΔV at departure and arrival in the departure year 2022 to 2030 for the time of flight ranging from 15.5 to 8.5 years.

By comparing the results with different TOF ranging from

15.5 to 11.5 years, the total ΔV increases gradually with the increase in departure years from 2022 to 2030. Similarly, for the TOF 9.5 and 8.5 years, the total ΔV decreases gradually with the increase in departure year from 2022 to 2030. Whereas for the TOF 10.5 years, it is observed that the ΔV presents a variation throughout the departure year 2022-2030 timeframe with a slight rise in the year 2024.

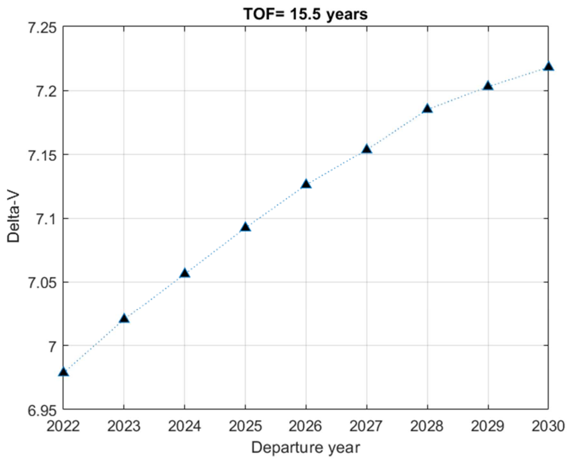

Figure 11. *ΔV variation vs departure year for TOF= 15.5 years.*

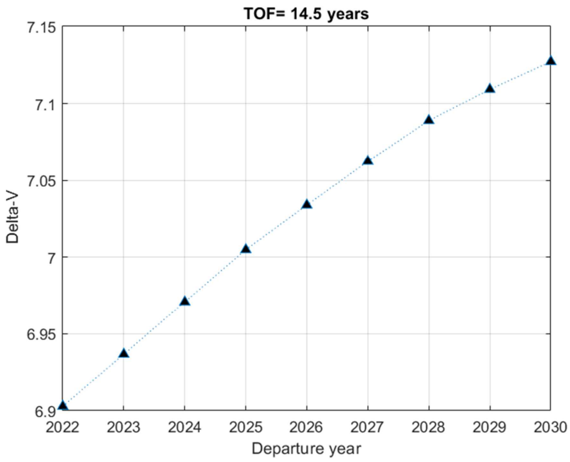

Figure 12. *ΔV variation vs departure year for TOF= 14.5 years.*

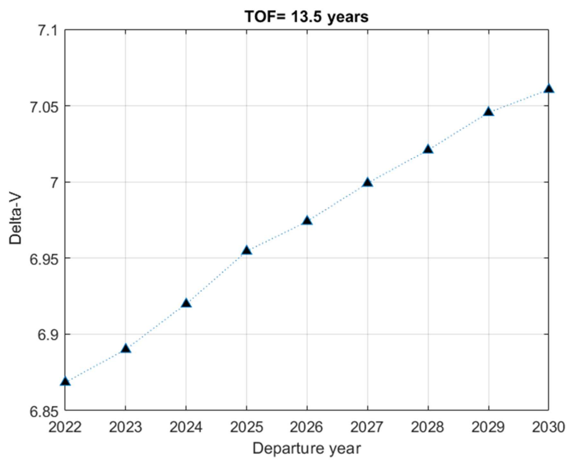

Figure 13. *ΔV variation vs departure year for TOF= 13.5 years.*

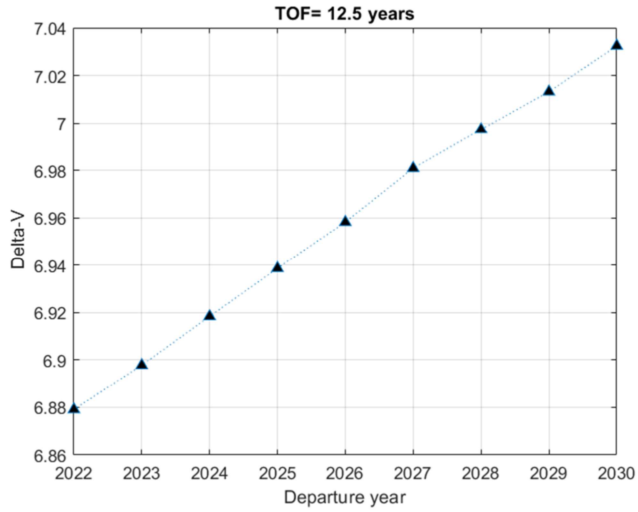

Figure 14. *ΔV variation vs departure year for TOF= 12.5 years.*

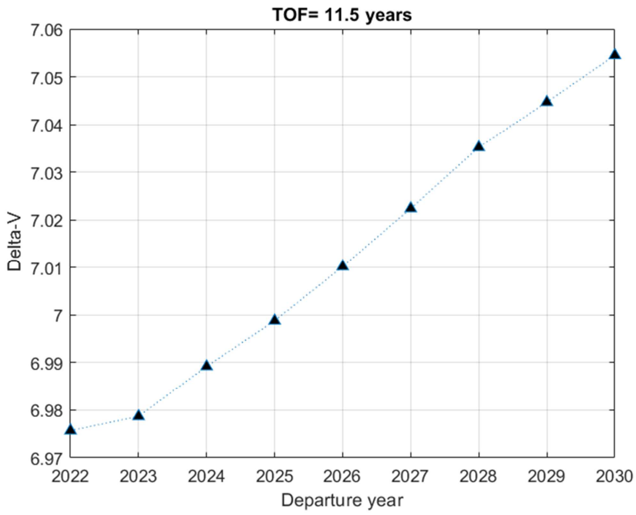

Figure 15. *ΔV variation vs departure year for TOF=11.5 years.*

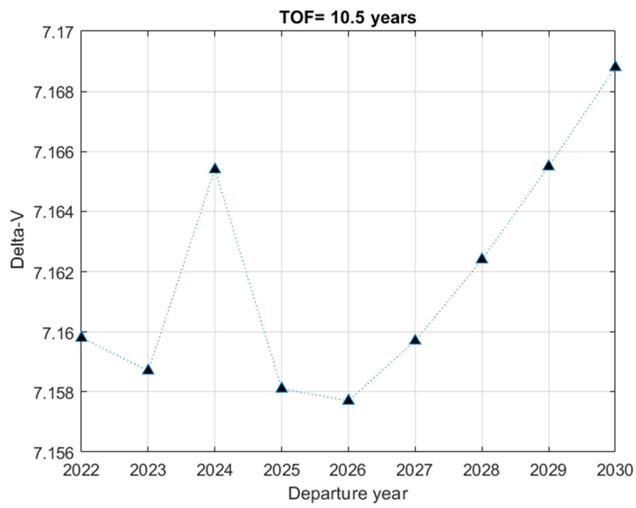

Figure 16. *ΔV variation vs departure year for TOF=10.5 years.*

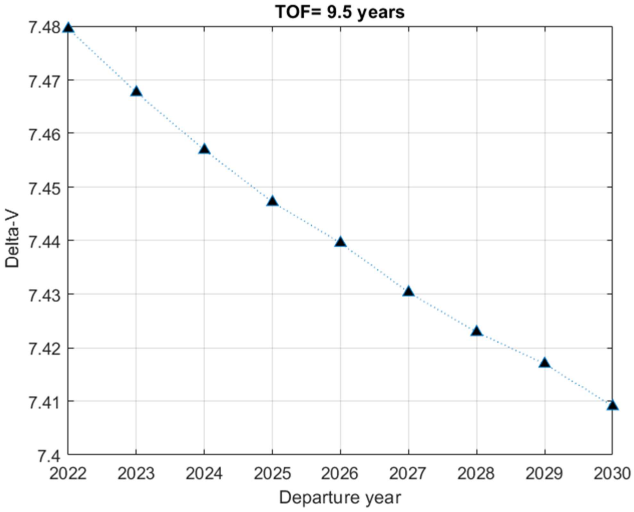

Figure 17. *ΔV variation vs departure year for TOF=9.5 years.*

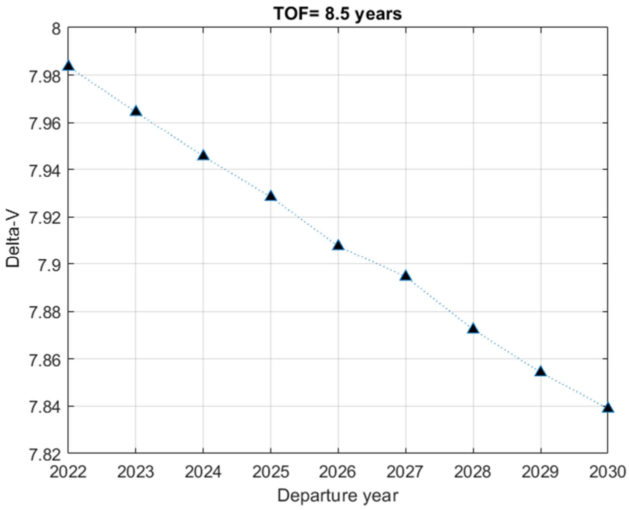

Figure 18. *ΔV variation vs departure year for TOF=8.5 years.*

Table 2. *Optimum ΔV for the TOF ranges from 16.5 to 8.5 years for the departure year 2022-2030.*

Departure date	Arrival date	TOF (years)	V_∞ at departure (km/s)	V_∞ at arrival (km/s)	ΔV at departure (km/s)	ΔV at arrival (km/s)	Total ΔV (km/s)
27/07/2022	21/01/2039	16.5	11.9566	4.7956	6.0091	1.0727	7.0818
22/07/2022	16/01/2038	15.5	11.8213	4.7784	5.9098	1.0689	6.9787
17/07/2022	11/01/2037	14.5	11.7035	4.823	5.8239	1.0791	6.9029
12/07/2022	07/01/2036	13.5	11.615	4.9509	5.7595	1.109	6.8685
09/07/2022	04/01/2035	12.5	11.5501	5.1871	5.713	1.1661	6.8791
04/07/2022	30/12/2033	11.5	11.5494	5.5686	5.7121	1.2637	6.9757
22/07/2026	17/01/2037	10.5	11.6205	6.0435	5.7636	1.3942	7.1577
08/08/2030	04/02/2040	9.5	11.7051	6.6789	5.8251	1.5839	7.409
07/08/2030	03/02/2039	8.5	11.8353	7.6842	5.92	1.9188	7.8389

Table 2 gives the V_∞ and ΔV at departure and arrival, and the total ΔV for the TOF from 16.5 to 8.5 years for the departure years 2022–2030. From these results, it is observed that the hyperbolic excess velocity and hence ΔV at departure, initially decrease and then increases for the departure years 2022– 2030.

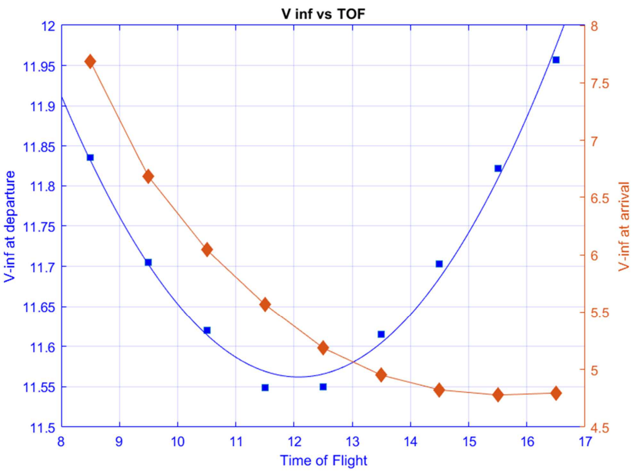

Figure 19. *Effect of V_∞ vs TOF.*

However, the hyperbolic excess velocity and hence ΔV at arrival, steadily grow with the reduction in flight duration. At 13.5 years of flight time, the ΔV drops to 6.86 km/s and then progressively increases as the flight time decreases. The total ΔV acquired via direct trajectory transfer is analyzed to be the lowest for 13.5 years of TOF. Figure 21 displays the simulation of the trajectory using FreeFlyer. The departure date and TOF of the spacecraft during its journey to Uranus are August 08, 2030, and 8.5 years.

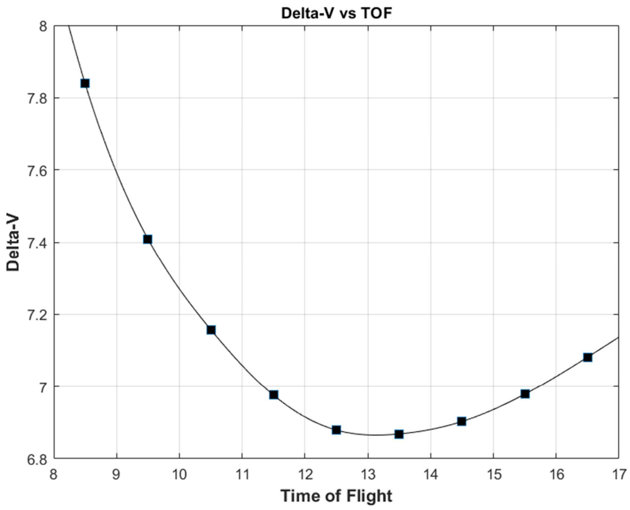

Figure 20. *Effect of ΔV (km/s) vs TOF in years.*

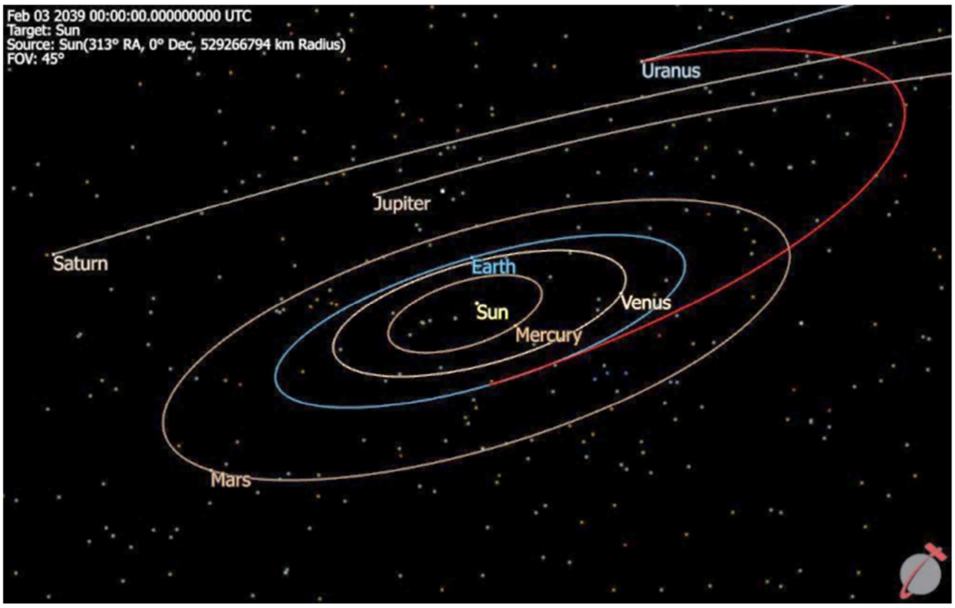

Figure 21. *Direct transfer trajectory towards Uranus using FreeFlyer.*

4. Conclusions

In this analysis, possible direct transfer trajectories from Earth to Uranus are discussed for launch years spanning 2022 to 2030. The minimum ΔV has been taken for different TOF varying from 15.5 – 8.5 years. Additionally, a Hohmann transfer is performed to get maximum TOF from Earth to Uranus. This is fixed as the upper limit of Lambert's trajectory for this mission. From the results, for the departure years 2022 – 2030, the hyperbolic excess velocity first reduces and then increases, whereas the hyperbolic excess velocity at arrival increases gradually with the reduction in TOF.

For each departure year, ΔV is minimum in July or August. The ΔV for the direct transfer trajectory to Uranus is observed to be in the range of 6.87 – 7.98 km/s. It first reduces to 6.86 km/s at 13.5 years and then increases gradually with the reduction in TOF. Results show that for 13.5 years of TOF the total ΔV obtained using the direct transfer is minimum for the mission, that is, 6.8685 km/s.

The future scope of this work is to investigate the arrival declination which is a significant design variable for the Uranus mission. Also, the different post-capture orbits can be explored to analyze the moon tour problem of Uranus.

Nomenclature

TOF	Time-of-flight, years
M	Mean anomaly, deg
E	Eccentric anomaly, deg
ω	Argument of perigee, deg
Ω	Right ascension of ascending node, deg
a	Semimajor axis, km
e	Eccentricity
i	Inclination, deg
a_e	Semimajor axis of Earth, km
P_1	Position of the vehicle at distance r_1
SOI	Sphere of influence
V_E	Orbital velocity of Earth, km/s
V_U	Orbital velocity of Uranus, km/s
μ_{sun}	Gravitational constant of Sun, km^3/s^2
r_1	Position vector of departing planet, km
r_2	Position vector of arriving planet, km
V-inf, V_∞	Hyperbolic excess velocity
Δt	Transfer time, years
a_u	Semimajor axis of Uranus, km
P_2	Position of the vehicle at distance r_2

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